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MAXIMUM RESPONSE RANGES OF NONLINEAR MULTI-STORY STRUCTURES SUBJECTED TO EARTHQUAKES

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ABSTRACT

The earthquake responses of a twenty-story nonlinear structural frame representative of a modern high-rise building were calculated. The structure was modeled by a two-dimensional frame with girders and columns having bilinear bending moment-end rotation hysteretic characteristics. In addition to hysteretic damping, viscous damping mechanisms were assumed. The earthquakes used were the El Centro (N-S) earthquake of May 18, 1940 and several pseudo-earthquakes having statistics similar to those of previously recorded earthquakes. Time history plots for displacements, bending moments, joint rotations, etc., at several stations in the structure are presented. A displacement envelope and a plot of the ductility factors, which measure the amount of yielding incurred, are also given. It was possible to identify certain behavior characteristics of the structural responses which appeared to be determined more by the properties of the structure than by the earthquake. For the series of pseudo-earthquakes used, a large range was found in the maximum values of the responses of the yielding structure. For example, the spread between extremum values of the maximum absolute displacements was 50 per cent of the arithmetic mean value. Statistics of the magnitudes of the displacements and ductility factors were compared with three common measurements of the strength of earthquake accelerograms. It was found that none of these three measurements correlated well with the trend of the maximum responses.

Introduction

Because of the comparative difficulty of obtaining the earthquake response of nonlinear multi-degree of freedom systems, most of the earthquake engineering effort in the past thirty or forty years has been directed towards investigating one degree of freedom systems. Often the resulting information was used to estimate the corresponding earthquake response of multi-degree of freedom structures. In recent years a great deal of effort has been successfully applied towards understanding the earthquake response of nonlinear multi-story structures (Penzien, 1960; Berg, 1961; Clough and Benuska, 1966; Giberson, 1967).

The earthquake response results for a nonlinear twenty-story structure are presented in this article in the form of time history response records, displacement envelopes and ductility factors. In order to compare the trend of the magnitudes of the structural responses with the trend of the earthquake strengths, several statistics are tabulated from which a number of conclusions are drawn.

The Structural Model

The model used is intended to be representative of modern high-rise buildings, having glass and other lightweight walls, such as are commonly being built today (1967). This structural model is characterized by the following assumed properties:

(1) The foundation is infinitely rigid.

(2) The structure is symmetric in plan view; hence, torsional deformation is neglected.

(3) The girders provide all of the stiffness of the floors and they are flexible (not infinitely rigid).
(4) The columns provide all of the stiffness of the walls.
(5) There are no shear walls; hence, the structure is of the moment-resisting type. This means that the structure resists deformation only by the moments developed at the ends of the girders and columns.
(6) Shear deflection in the girders and columns is neglected.

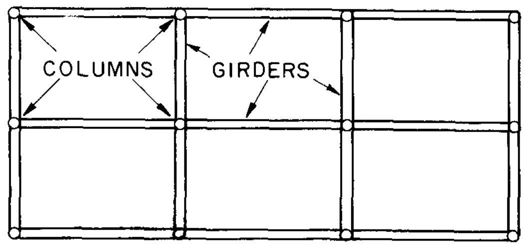

Fig. 1a Plan view of a moment-resisting frame

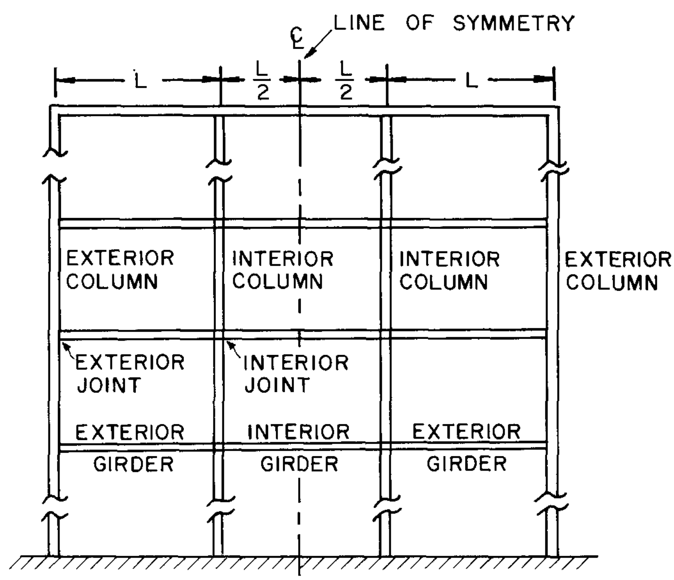

FIG. 1b. Side view of a moment-resisting frame.

(7) The girders and columns can yield at each end according to a bilinear bending moment-end rotation hysteresis loop.
(8) There is no relative deflection between joints within the same floor.
(9) There are no vertical deflections between floors since it is assumed that the structure is infinitely rigid in the vertical direction.
(10) All mass is concentrated at the floor levels.
(11) The mass of each floor moves horizontally only. Vertical motion is not considered because of property (9). Although the joints rotate, the rotational inertia associated with each joint is neglected.
(12) Gravitational effects are negligible for the base overturning moment.

Only one (two-dimensional) structural frame is analyzed at a time. In addition to the properties listed above, the structural frame studied also has the following:

a. It has three bays (four columns) and is symmetric with respect to the centerline as shown in Figure 1b.
b. The structural properties of the frame are taken to be representative of the three-dimensional structure shown in Figure 1a.

One frame having the above structural properties was used, the structural parameters being given in Figure 2. This frame was designed by R. W. Clough and K. L. Benuska using conventional static analysis techniques, taking into consideration static gravity loads plus lateral loads as specified by the Uniform Building Code in

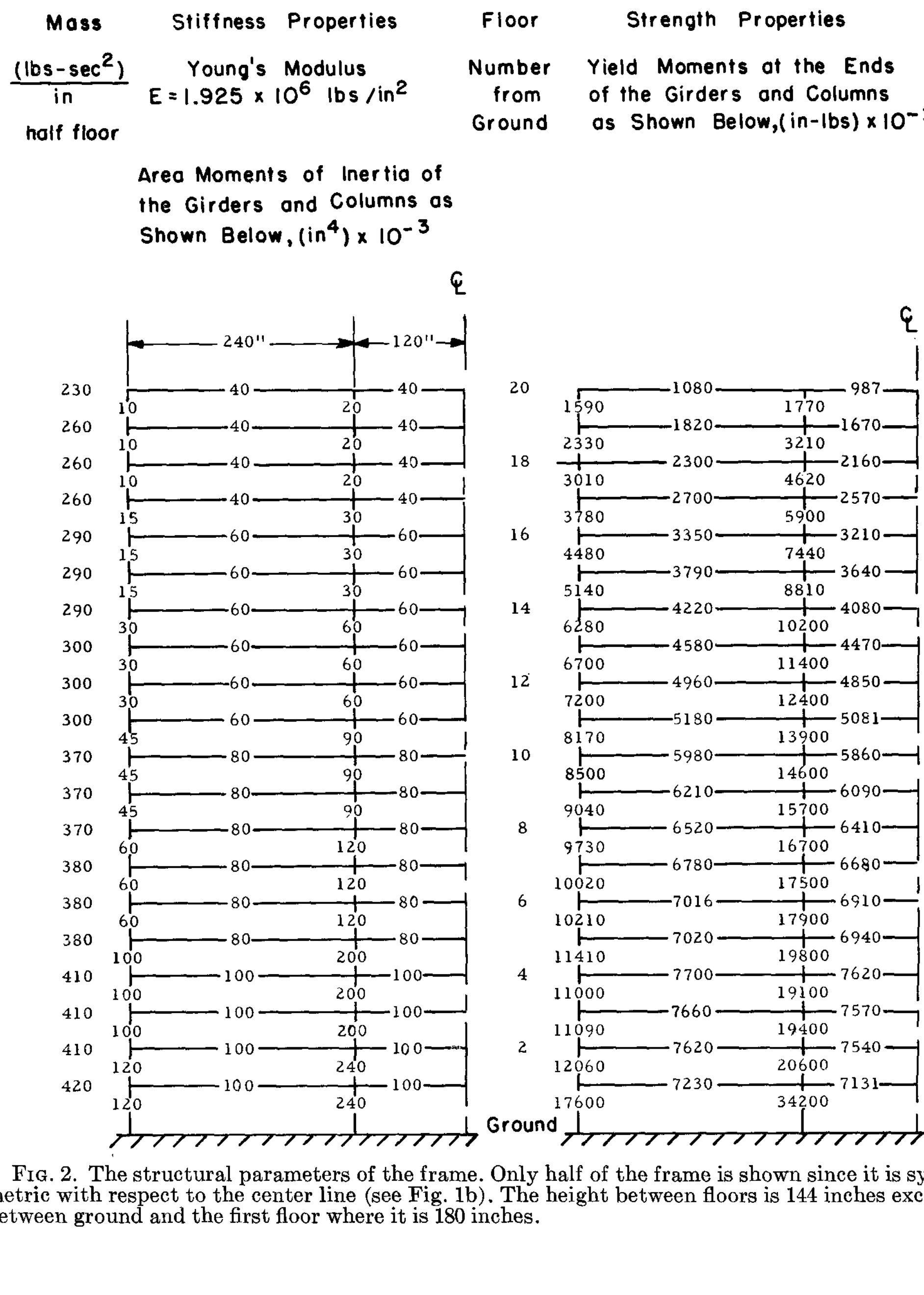

FIG. 2. The structural parameters of the frame. Only half of the frame is shown since it is symmetric with respect to the center line (see Fig. 1b). The height between floors is 144 inches except between ground and the first floor where it is 180 inches.

effect in 1965. An advantage of using this frame was that independently obtained response results were available (Clough and Benuska, 1966) for comparison with results obtained using the present method of analysis for one combination of structure and earthquake parameters. Since the corresponding response results were the same, an additional check on the calculations has been provided.

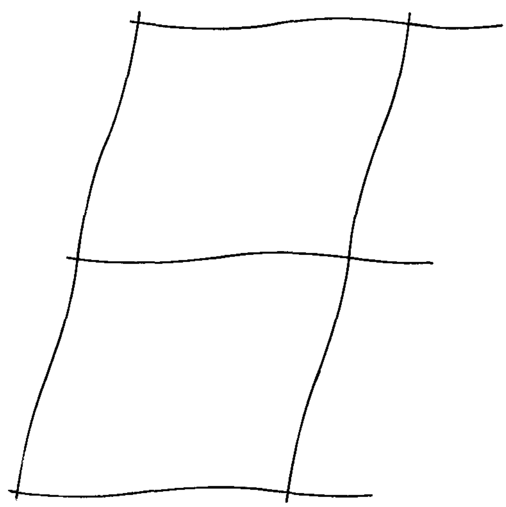

FIG. 3. Section of a typical moment-resisting frame in a deformed position.

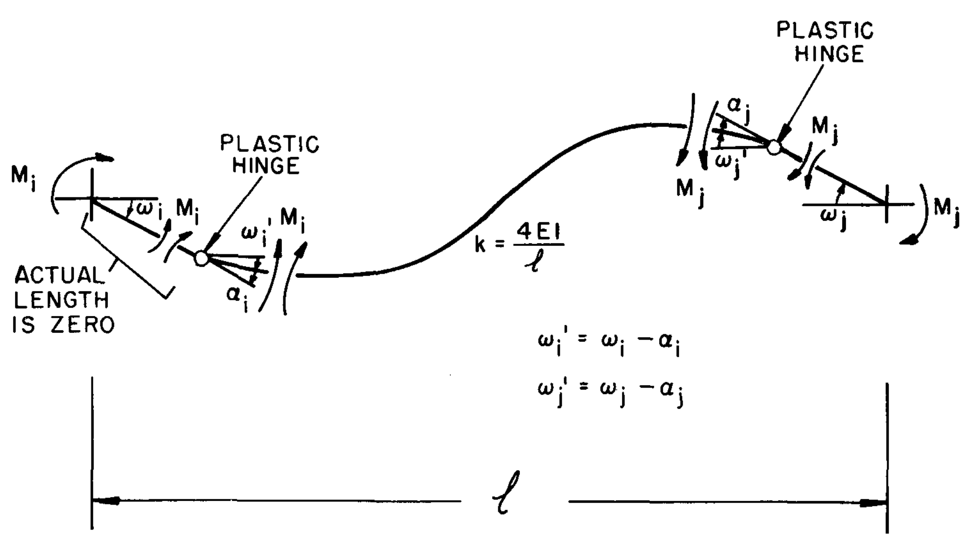

FIG. 4. The beam model with bilinear hysteretic characteristics at each end.

A section of a frame having the general structural properties enumerated above is shown in a deformed position in Figure 3 and the nonlinear beam model used to represent each girder and column in the frame is depicted in Figure 4 (Giberson, 1967). Although bilinear hysteretic bending moment-end rotation characteristics were used in this study, this beam can also be used to model curvilinear hysteretic loops since the plastic hinges at the ends are independent of each other (Giberson, 1967). In

Figure 4 the following nomenclature is used (similarly for end (j)):

$k = 4EI/l$	stiffness of beam for small end rotations;
M_i	bending moment at end (i);
ω_i	rotation of end (i);
α_i	incurred plastic angle at end (i); and
$\omega_i' = \omega_i - \alpha_i$	rotation of the central section of the beam at the plastic hinge at end (i).

Damping

The damping mechanism used in the studies presented here is called "mass propor-

TABLE 1

Earthquake	Fraction Critical Damping in Frame	rms Scale Factor (with respect to El Centro)	Max. Abs. Acceleration of Earthquake (fraction of grav.)	Spectrum Intensity $SI_{0.0}$	Max. Abs. Displacement (inches)	Max. Abs. Interfloor Displacement (inches)	Maximum Ductility Factors		
							All girders	Interior column	Exterior column
El Centro (N-S) 18 May 1940	0.10	1.0	0.316	8.94	11.52	1.29	4.44	2.99	0.83
Pseudo-earthquake No. 1	0.10	1.15	0.246	11.45	8.98	0.98	3.58	1.74	0.71
Pseudo-earthquake No. 2	0.10	1.18	0.267	13.23	14.43	1.31	5.00	3.11	0.81
Pseudo-earthquake No. 4	0.10	1.26	0.267	12.21	10.96	0.93	3.96	2.02	0.80
Pseudo-earthquake No. 5	0.10	1.19	0.271	12.25	9.69	0.91	3.63	2.23	0.76
Pseudo-earthquake No. 6	0.10	1.22	0.313	10.96	8.91	0.99	4.01	2.58	0.78
Pseudo-earthquake No. 6	0.05	1.22	0.313	10.96	10.28	1.08	4.32	2.91	0.80
Pseudo-earthquake No. 7	0.10	1.20	0.328	12.06	8.52	0.97	3.75	2.53	0.79

Notes:
These pseudo-earthquakes were chosen at random.
In all cases the bilinearity is 0.05.
Max. Abs. = Maximum Absolute.

tional" viscous damping. The expression for this type of damping for an N-floor structure is, in incremental matrix form,

$$\gamma M \boldsymbol{\Delta}\dot{\mathbf{u}}$$

where

γ	scalar constant,
M	diagonal mass matrix ($N \times N$), and
$\boldsymbol{\Delta}\dot{\mathbf{u}}$	vector ($N \times 1$) of the incremental velocities of the floors relative to the ground.

An additional damping mechanism which could have been used is called "stiffness proportional" viscous damping which corresponds to interfloor energy dissipation caused by walls and partitions cracking and rubbing together. A convenient approximation for this type of damping is the expression:

$$\beta K \Delta \dot{\mathbf{u}}$$

where

- β scalar constant
- K tri-diagonal stiffness matrix ($N \times N$), and
- $\Delta\dot{\mathbf{u}}$ vector ($N \times 1$) of the incremental velocities of the floors relative to the ground.

These two damping mechanisms together comprise what is termed Rayleigh damping for this structure.

Since mass proportional damping only was used in the structural model, the higher frequency responses are not so effectively damped as are the lower ones; consequently, there is a limitation on the model for the representation of those responses which exhibit significant high frequencies. Although stiffness proportional damping was not

TABLE 2
NOMENCLATURE USED IN THE TIME HISTORY PLOTS*

TOTAL ACCEL 17F	The total horizontal acceleration of the 17th floor
HORIZ DISP 4F	The horizontal displacement of the 4th floor relative to ground
OVRTURNG MOMT-BASE	The overturning moment at the base of the three-bay structure
SHEAR FORCE 18-19F	One-half of the interfloor shear force between the 18th and 19th floors (from only the two columns used in the analysis)
INT-FL DISP GD-1F	The interfloor displacement between the ground and the first floor
JNT ROT'N 17F-XC	The rotation of the joint formed by the intersection of the exterior girder of the 17th floor and the exterior column line
BND MOMT 17F-XG-XC	The bending moment at station (e), Figure 5
PL ANGLE 4F-XG-XC	The plastic angle at station (f)
PL INDEX 4F-XG-XC	The plastic index at station (f). This index indicates when yielding occurs and the direction of incremental yielding
BND MOMT 20F-IC-BT	The bending moment at station (d)
MOD JNT ROT 20F-IC	The rotation of the joint formed by the intersection of the 20th floor and the interior column line modified to include the effect of the interfloor shear angle

* The unit system used is "in-lbs-sec" with the joint rotations given in radians.

used in these analyses, no additional computational difficulty would have been involved in introducing it.

Response Results

The input accelerograms and the parameters used are described below with a summary given in Table 1. In addition to the El Centro (N-S) earthquake of May 18,

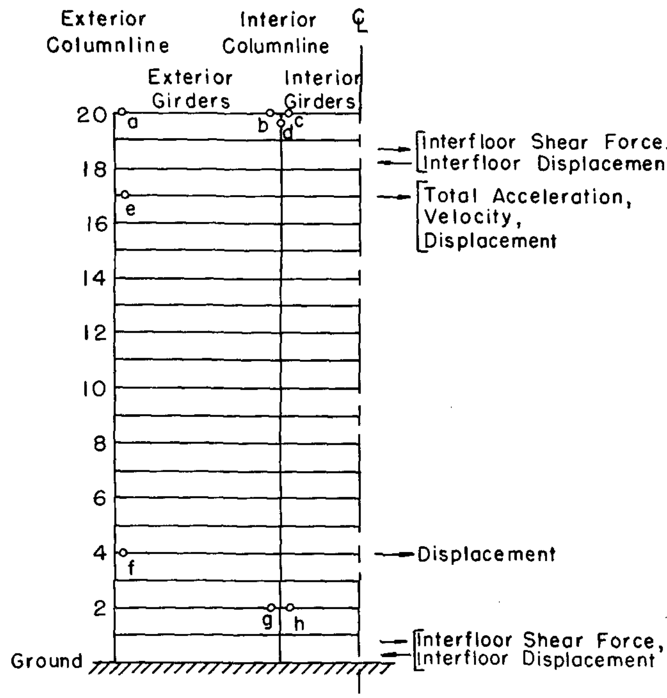

FIG. 5. o: Stations for which the bending moment, joint rotation (or modified joint rotation), plastic angle, and plastic index are plotted.

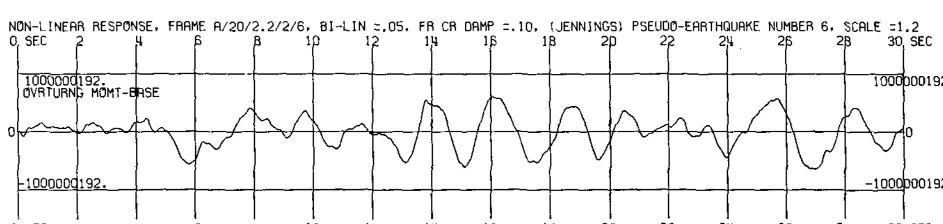

FIG. 6

1940, six of the pseudo-earthquakes generated from a random process by Paul C. Jennings (1963) were used. These pseudo-earthquakes have statistics similar to those for previously recorded strong-motion accelerograms of real earthquakes. The pseudo-earthquake accelerograms are more nearly uniform in amplitude over time than are the real ones. To obtain approximately the same amount of yielding in the structure for the pseudo-earthquakes as occurred for the El Centro earthquake, they were scaled so that the rms value of each accelerogram is approximately 1.2 times that of El Centro, the actual rms values being given in Table 1. In all cases the bilinearity (the ratio of the second slope to the first in the hysteresis loop) of the structure was 0.05. The fraction of critical damping in the fundamental mode was 0.10 in all studies except one which used 0.05 fraction of critical damping in order to observe the effect of damping on the response. The fundamental period of the structure was 2.21 sec.

The modes of vibration referred to herein are those of the linear system before yielding has occurred. For the nonlinear system, if the nonlinearities are not large, the response can be approximated at any point in time by a series of terms having the

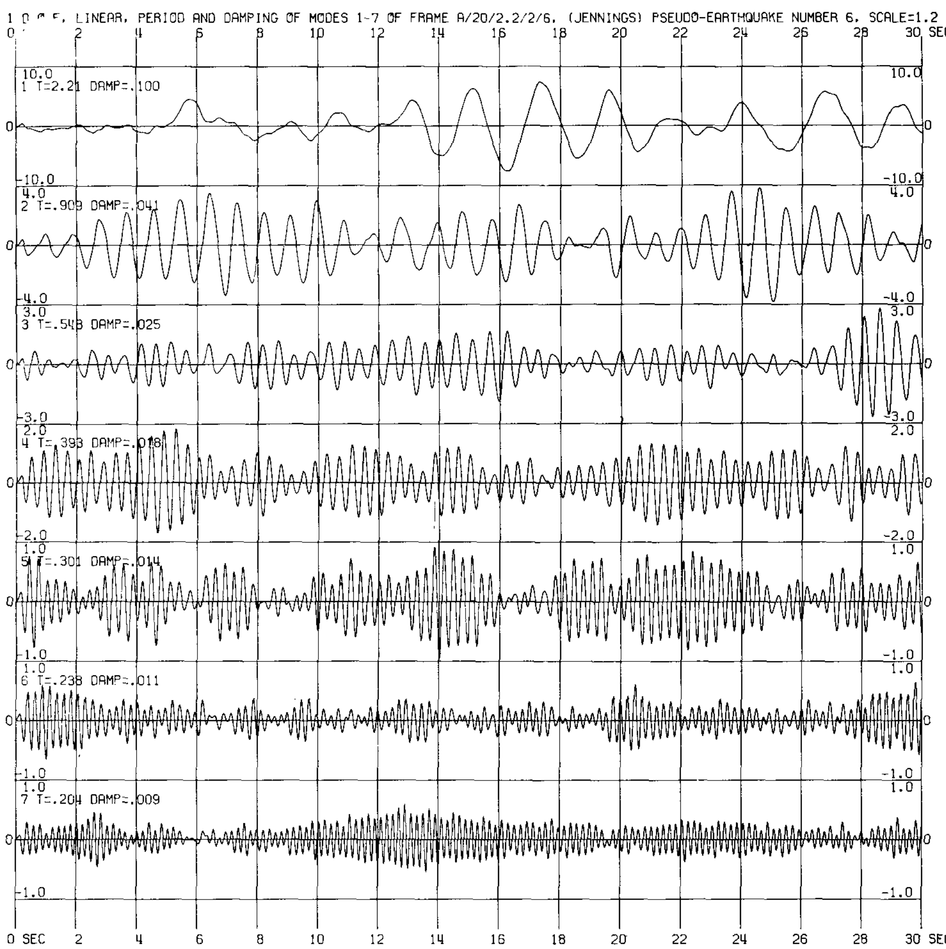

FIG. 7

mode shapes of the linear system. This decomposition with respect to the linear modes permits interesting comparisons to be made between the linear and the nonlinear response of the structure, as is shown below.

The response of the nonlinear structure subjected to the entire 30 seconds of pseudo-earthquake number 6 with 0.10 fraction of critical damping (mass proportional) in the fundamental mode is presented in detail here. The plots given are time histories of bending moments, interfloor shear forces, joint rotations, total accelerations, displacements and interfloor displacements for a number of stations in the structure. The nomenclature used in the plots is given in Table 2 and the stations are indicated in

Figure 5. In addition to the time histories, the displacement envelope and the ductility factors are shown. Furthermore, the displacement responses (in inches) of seven one-degree of freedom linear oscillators subjected to the same excitations are presented in Figure 7. These oscillators model the first seven modes of the linear structure with respect to periods and damping values. For example, the oscillator which corresponds to mode 1 has a period $T = 2.21$ sec and fraction of critical damping 0.100. In Figure 7 the response of this oscillator is labeled

$$1\ T = 2.21 \text{ DAMP} = .100.$$

The time history of the displacement of the 17th floor of the nonlinear structure is shown in Figure 8. By comparing it with the response of the linear oscillator corresponding to the first mode of the linear frame, henceforth called the first modal os-

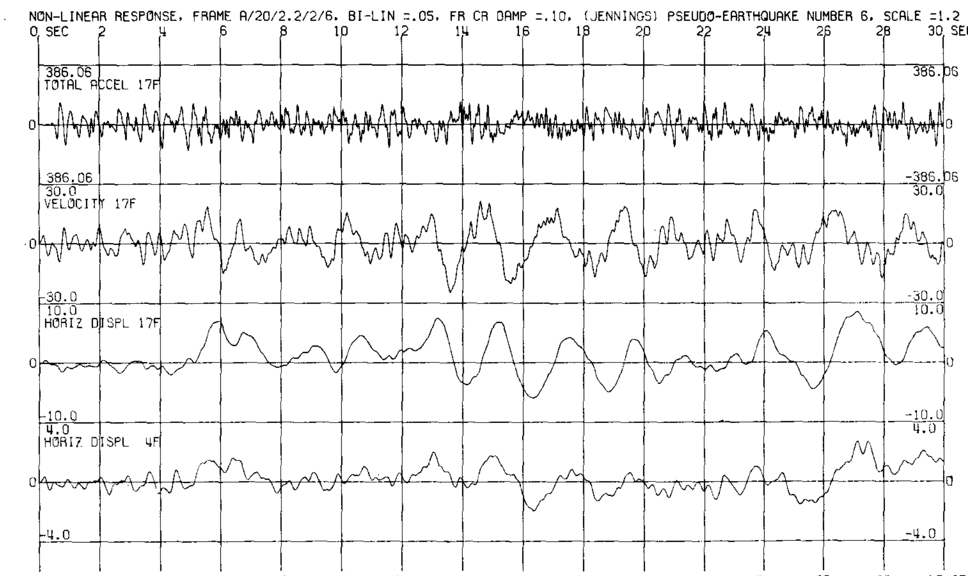

FIG. 8

cillator, it is seen that the two are closely correlated. This is in accordance with analytical results for linear cantilever beam models of multi-story structures (Giberson, 1967) which indicate that the horizontal displacements in the upper portion of the structure are expected to be dominated by the first few modes, primarily the fundamental.

By design, the distribution of yield bending moments is such that throughout most of the structure the girders yield at a lower level than the neighboring columns. For all of the earthquake excitations used, the interior columns of the structure yielded in the upper two or three floors only and the girders spent at most 10 per cent of the time in yield. Consequently, the time histories of the gross responses of this particular structure, such as the displacements, should be highly correlated with those of the linear structure. If the yield moment distribution were changed so that more of the columns yielded, a lower correlation with the response of the linear system would probably occur.

The overturning moment at the base, Figure 6, is observed to be closely correlated with the displacement at the 17th floor and with the response of the first modal oscillator (with opposite sign). This correlation should exist for the following reason: when a floor is displaced, shear forces develop in the girders which appear as forces in the axial direction in the columns. Only when many of the floors are displaced in the same direction, such as would occur in the fundamental mode, do the axial column forces add together to cause a significant resultant moment at the base.

A comparison of the 4th floor displacement with the 17th floor displacement, Figure 8, indicates that higher frequency response of the structure influences the displacement of the 4th floor more than it influences the displacement of the 17th floor. This is in agreement with studies of cantilever beam models of structures referenced above.

In addition to a fundamental frequency component, higher frequency response is observed in the plot of the velocity (relative to ground) of the 17th floor. This is ex-

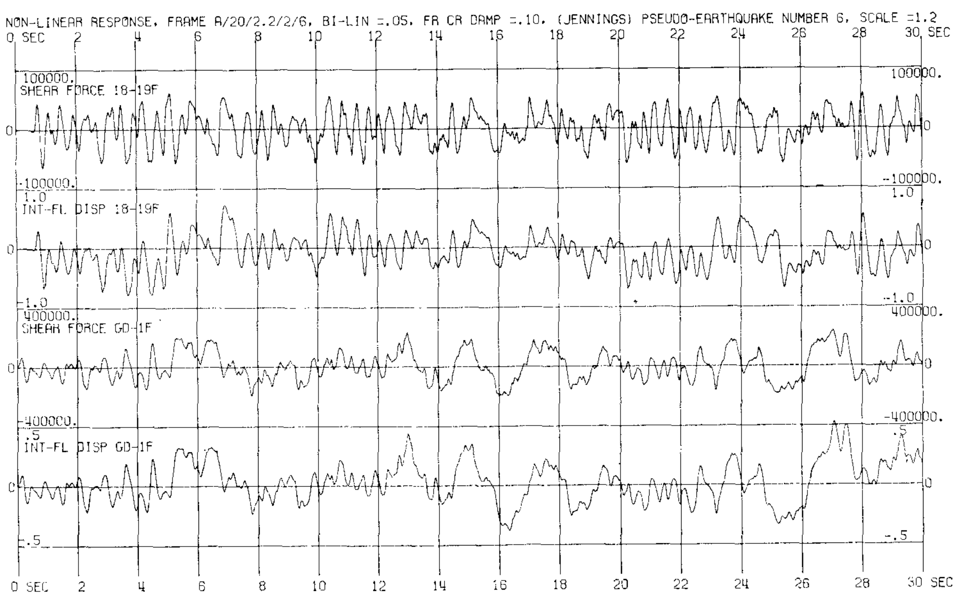

FIG. 9

pected since the velocity is a first derivative of the displacement and differentiations characteristically tend to amplify higher frequencies.

In the plots in Figure 9 it is seen that the shear force and interfloor displacement between the ground and first floor have a strong fundamental frequency component. However, near the top of the structure, many frequencies are observed in the shear force and interfloor displacement records.

Time history plots of the bending moments, joint rotations, plastic angles, and plastic indices (Table 2) are shown in Figures 10, 11 and 12 for three stations in the structure. If the station is in a girder, the joint rotation is given; if the station is in a column, the modified joint rotation taking into account the interfloor displacement is plotted.

By making visual comparisons of the time-history plot of the bending moment at station (e), Figure 10, with the response of each of the linear modal oscillators, Figure 7, various correlations can be observed. Between 3 sec and 10 sec a strong correlation with the second modal response is observed; hence, it is concluded that the yielding during this time at this station is strongly dependent upon the second modal frequency component. Between 12 sec and 19 sec correlation with the first modal response occurs. Similarly, it is concluded that during this interval, the yielding is mostly caused by the fundamental frequency component. At times, the yielding is dependent upon combinations of frequency components since the correlation with the response of no one modal oscillator appears to be sufficient to cause yielding.

At station (f) in the 4th floor, Figure 11, the fundamental frequency component appears to be the largest contributor to the bending moment and, hence, is the most

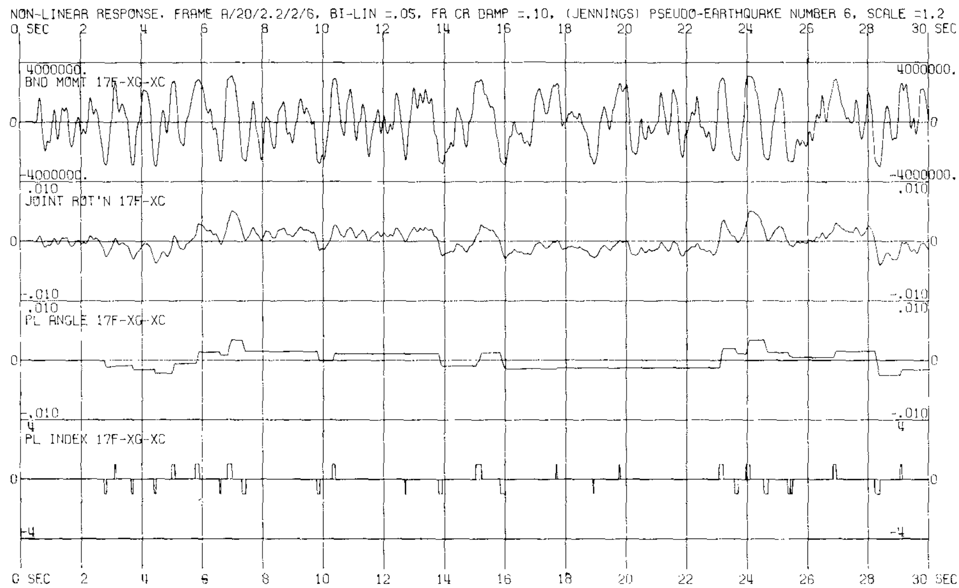

FIG. 10 Station (e)

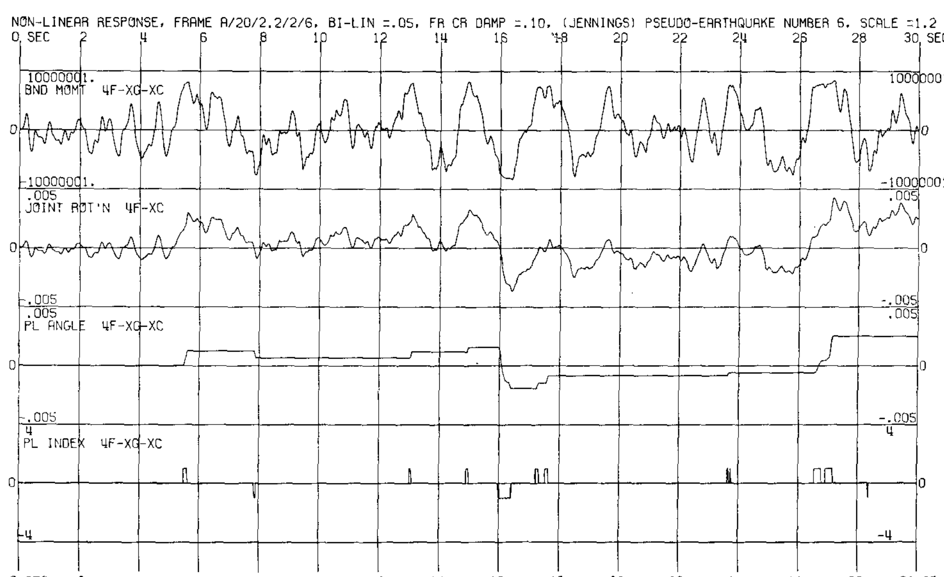

FIG. 11 Station (f)

important for yielding at this station. At times, however, the second and higher modal frequency components are observed to add to the fundamental frequency component, thereby contributing to the yielding. On the other hand, in the time history plots of the bending moment in the interior column at the 20th floor, Figure 12, many frequencies are observed. It is not obvious that any one frequency dominated the bending moments or the yielding in the 20th floor. Furthermore, the times at which yielding occurred in

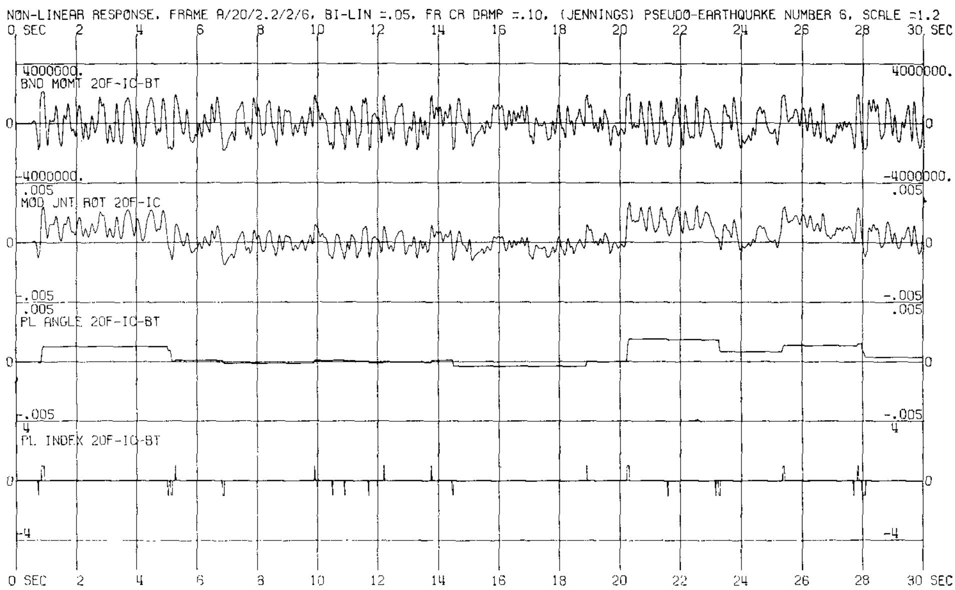

FIG. 12 Station (d).

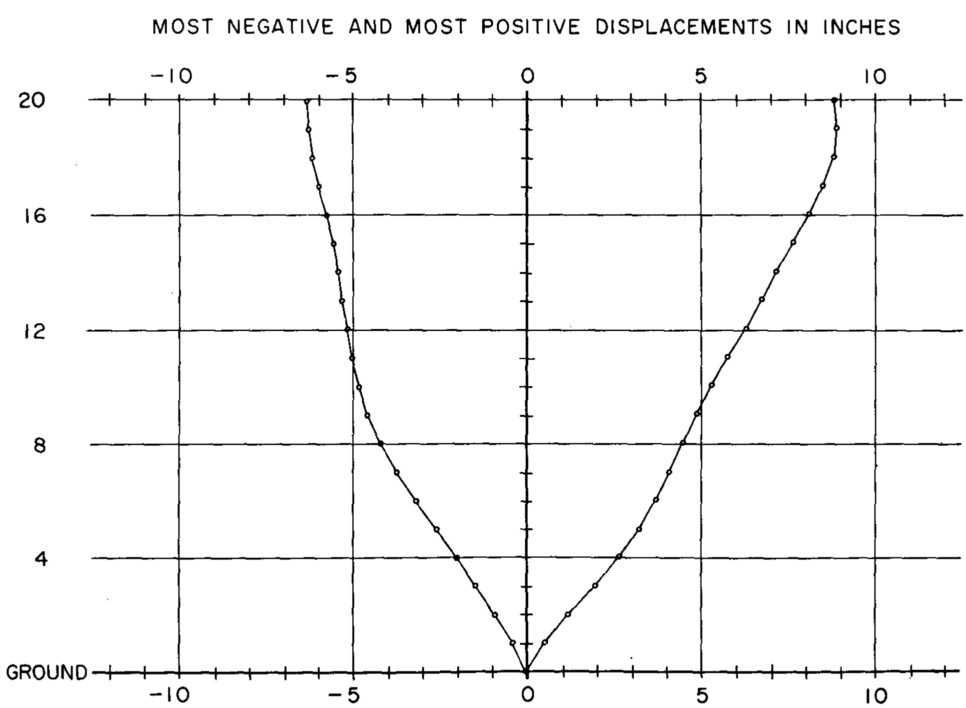

FIG. 13

the 20th floor are different from those at which yielding occurred in either the 4th or the 17th floor.

In Figure 13 the displacement envelope is given. Although the points are connected together, this does not necessarily imply that either all of the most positive displacements or all of the most negative displacements occurred at the same time. In some cases, three or four times were represented for either side of the envelope. However, in many cases, including the particular case presented here, all of the floors did reach their maximum displacements at approximately the same time indicating the importance of fundamental frequency response to the displacements. The 20th floor (roof) usually, but not always, was the floor which incurred the maximum absolute displacement of any floor during excitation. As indicated in Table 1, for the various studies using 0.10 fraction of critical damping in the frame, the maximum absolute displacements ranged between 8.5 and 14.4 inches.

In the lower portion of the structure the yield moments in the girders are lower than

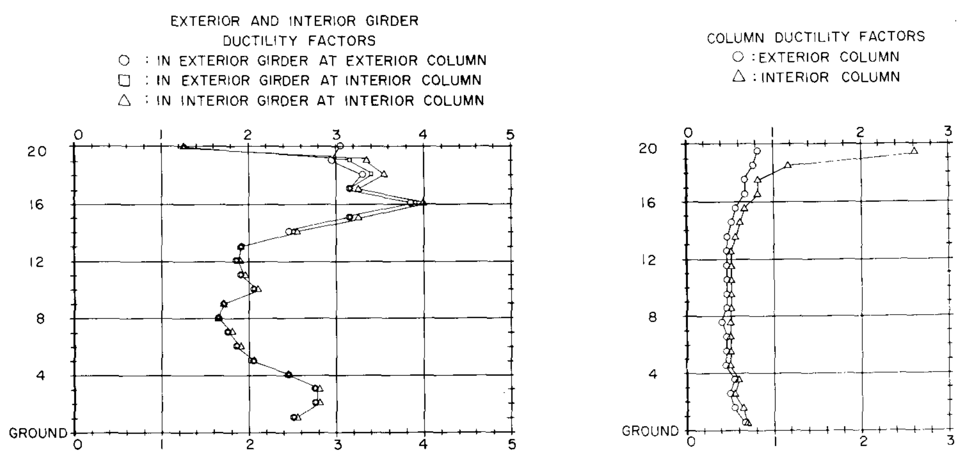

Fig. 14a. Fig. 14b.

FIG. 14

those in the neighboring columns (Figure 2); consequently, the columns remained linear and the girders yielded. Near the top of the structure the yield moments in the girders approximate the yield moments in the neighboring columns. As a result, in this portion of the frame, both the girders and the columns yielded. This can be seen in the plots of the ductility factors for the individual girders and columns in Figure 14a and Figure 14b, respectively. The ductility factors, μ, greater than 1.0 indicate that yielding occurred and are calculated according to the equation (Giberson, 1967)

$$\mu = 1 + \left| \frac{\alpha}{\omega_y + \dfrac{\alpha}{(1 - p)}} \right|_{\max}$$

where

p = ratio of the second slope to the first slope of the bilinear hysteresis loop;

ω_y = end rotation angle corresponding to incipient yielding of the beam of Figure 4 when loaded symmetrically in the directions shown; and

α = incurred plastic angle at an end of the beam.

When yielding did not occur, the ductility factors are less than 1.0 and are given by the maximum absolute values of the ratios of the incurred bending moments to the corresponding yield bending moments. For the girders, the ductility factor for each end is shown, but for the columns, only the maximum value that occurred at

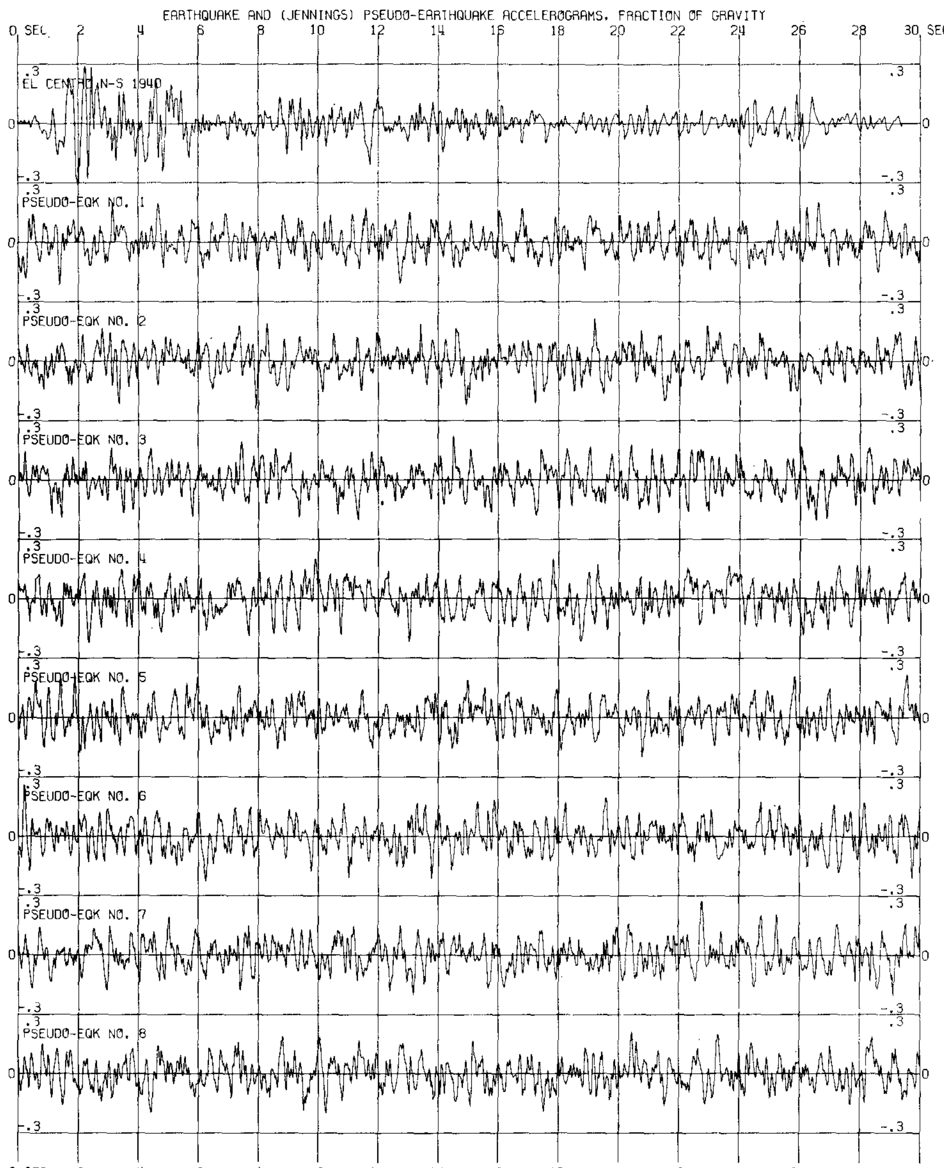

FIG. 15. Earthquake accelerograms. Scale = 1.0.

either end is plotted. The maximum values of the ductility factors resulting from each earthquake are presented in Table 1. For the pseudo-earthquakes, with 0.10 fraction of critical damping in the frame, the maximum values of the ductility factors for the girders ranged from 3.58 to 5.0. For the exterior columns, which did not yield, the maximum values ranged from 0.71 to 0.81. For the interior columns, which yielded in the upper two or three floors only, the maximum values were between 1.74 and 3.11.

This range in the maximum values of the structural responses appears to result from statistical variation. Such a range might be expected because the exact time histories of the earthquake accelerograms differ and because there is some variation in the statistics of the properties of the accelerograms.

In order to observe the effect of viscous damping on the response of this frame, one study using pseudo-earthquake number 6 with 0.05 fraction of critical damping in the fundamental mode was made. The summary of these response results tabulated in Table 1 indicates that when the fraction of critical damping was decreased from 0.10 to 0.05, the maximum absolute values of the various responses increased: the displacement by 15 per cent; the interfloor displacement by 9 per cent; the ductility factors in the girders by 8 per cent, in the interior columns by 12 per cent, and in the exterior columns (moment ratio) by 3 per cent.

Relationships between Structural Responses and the Strength of an Earthquake

Three common measurements of the strength of an earthquake are the following:

(1) The maximum absolute value of the ground acceleration during the earthquake;
(2) The rms value of the acceleration of the earthquake:

$$\left[\frac{1}{t'}\int_0^{t'} [\ddot{y}(t)]^2\, dt\right]^{\frac{1}{2}}$$

where t' is the length of the earthquake; and
(3) The spectrum intensity of the earthquake: the spectrum intensity SI_ξ is defined as the area under the velocity spectrum curve, S_{vel}, from period $T = 0.1$ to $T = 2.5$ for a given damping factor, ξ, and earthquake excitation of duration t' (Housner, 1959):

$$SI_\xi = \int_{T=0.1}^{T=2.5} S_{\mathrm{vel}}\left(\xi, \frac{2\pi}{T}, t'\right) dT.$$

The numerical values of these three strength measurements of the earthquakes used to excite the nonlinear structure are listed in Table 1. Note that the spectrum intensities are given for $\xi = 0.0$.

In an attempt to find a means for predicting the magnitude of structural response, it is of interest to compare the earthquake strengths defined above with the magnitudes of the corresponding structural responses (Table 1). Unfortunately, it is seen that none of these three measurements correlate well with the trend of the maximum absolute values of either the displacements or ductility factors of the nonlinear structure.

The first two measurements, maximum absolute value and rms value of an earthquake accelerogram do not take any properties of the structure into consideration and, therefore, they should not be expected to accurately estimate the maximum absolute value of the various responses of a multi-story structure. However, if the same earthquake is used with two different amplitude scale factors, the response corresponding to the larger scale factor can be expected to be larger.

The period range chosen for the spectrum intensity includes many of the natural periods of typical multi-story structures, but because the modes of a particular structure are not specifically considered, the spectrum intensity should not be expected to accurately predict the amplitude of the response of that structure. Nevertheless, the spectrum intensity may be indicative of the amplitude of the average response of a number of structures taken together.

From these results, it appears necessary to take the time dependence of the various modes into consideration in order to accurately predict the maximum values of the various responses.

Conclusions

(1) For the particular earthquake-excited, nonlinear structure studied here, characteristic patterns of frequency content occurred in the various structural responses. Furthermore, by comparison with the linear modal oscillator responses, the dominant frequencies in the nonlinear structural responses were observed to approximate the modal frequencies of the linear structure. It is expected, however, that with increasing motion in the nonlinear range, the resemblance of these characteristic frequency patterns to those of the linear structure will decrease.

(2) The nonlinear structure with 0.10 fraction of critical damping in the fundamental mode was subjected to the entire El Centro (N-S) earthquake of May 18, 1940 and to six pseudo-earthquakes. The characteristic patterns of response behavior noted are summarized below:

(a) For the overturning moment at the base, the fundamental frequency dominates.
(b) For the horizontal displacements in the upper portion, the fundamental frequency dominates; in the lower portion, the fundamental frequency is important, but the second and third modal frequencies are also significant.
(c) For the interfloor displacements in the upper portion, at least the first eight-modal frequencies may be important, while in the lower portion most of the response is accounted for by the first four-modal frequencies. The fundamental frequency is of minor importance near the roof, but its importance increases approaching the base.
(d) For the total accelerations, many frequencies are observed. This is partly a consequence of the relatively low damping of the higher frequencies.
(e) The distribution of the girder ductility factors are similar for the different earthquakes used. Column ductility factor distributions were also similar for the different earthquakes.

Because these results are consistent for all of the earthquakes used, it is believed that the characteristic patterns of response behavior and the shapes of the ductility factor distributions are a function more of the structure than of the earthquake.

(3) The frequency components of a given nonlinear structural response are excited in different amounts and at different times during an earthquake. Because the interaction between frequency components is so important to the maximum absolute value of a structural response, this value can differ considerably from one earthquake to another, even though statistics of the earthquakes may be similar.

(4) Comparing maximum responses of the yielding structure for the series of pseudo-earthquakes having similar statistics, the spreads between the extremum values were found to be the following percentages of the corresponding arithmetic mean values:

(a) maximum absolute displacements	50 per cent
(b) maximum absolute interfloor displacements	40 per cent
(c) girder ductility factors	35 per cent
(d) interior column ductility factors	55 per cent
(e) exterior column ductility factors	15 per cent

(5) The effect of decreasing the fraction of critical damping from 0.10 to 0.05 in the

fundamental mode of this particular structure excited by pseudo-earthquake number 6 is to increase the maximum values of the displacement response by 15 per cent of the ductility factors by approximately 8 per cent.

Acknowledgment

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